The last installment of this blog introduced the concept of arbitrage within the setting of mathematical finance and discussed the two pricing models: the Capitol Asset Pricing Model (CAPM) and the Arbitrage Pricing Theory (APT). Both of these models depend using a linear model for setting the price of some asset relative to some other asset in order to look for arbitrage opportunities. The asset can be a single financial vehicle, like a stock or bond, or, as is usually the case, it is some collection (a portfolio) of such simple assets.

Mathematically, the linear model that encompasses both the CAPM and APT is given by

where:

* is the expected rate of return of the asset in question,
* is the rate of return if the asset had no dependence on the identified macroeconomic factors (free rate of return),
* is the sensitivity of the asset with respect to the ith macroeconomic factor, and
* is the additional [risk premium](https://en.wikipedia.org/wiki/Risk_premium) associated with the ith macroeconomic factor with being the actual risk premium.

Previously, the

As in most things, it is much easier to understand this model with a concrete example (derived from Hayes’s article). Consider an asset that depends on the following four macroeconomic factors (i.e = 4):

* Gross domestic product (GDP) growth
* Inflation rate
* Gold prices
* and the return on the Standard and Poor’s 500 index

Historic data are typically analyzed, according to the available literature, via a linear regression. This process not only identifies the preceding four factors as the most important it also gives values for the sensitivity factor and the premiums for each. Assuming a free rate of return = 3%, the data conveniently present themselves in the following table:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Macroeconomic Factor | Sensitivity factor | Additional Premium | Risk Premium |  |
| GDP Growth | 0.6 | 7% | 4% | 2.4% |
| Inflation | 0.8 | 5% | 2% | 1.6% |
| Gold prices | -0.7 | 8% | 5% | -3.5% |
| S&P 500 | 1.3 | 12% | 9% | 11.7% |

Adding up each value in the last column and then adding the result to gives a value for the asset of = 15.2 %.

The list a APT macroeconomic factors commonly used include the ones listed above as well as corporate bond spread, shifts in the yield curve, commodities prices, market indices, exchange rates, and a host of others. Basically, any factor in the economy as a whole that effects all assets should figure in as there is no way to mitigate these risks by diversification.

In the above example, the parameters were assumed a priori. In his article [Arbitrage Pricing Theory: It’s Not Just Fancy Math](https://www.investopedia.com/articles/active-trading/082415/arbitrage-pricing-theory-its-not-just-fancy-math.asp), Elvin Mirzayev walks through how to simultaneously solve for the s to get what we are really after, the intelligently-derived expected return on the asset. (CFI’s [Arbitrage Pricing Theory](https://corporatefinanceinstitute.com/resources/knowledge/finance/arbitrage-pricing-theory-apt/) has a similar example that complements the previous presentation – financial gurus aren’t often clear in their explanations and having multiple sources helps.) Once that is obtained, it is compared to the offered rate and, when the two differ sufficiently, the asset is ripe for arbitrage.

The Wikipedia article on APT and Mirzayev’s piece discuss the importance of developing a portfolio of assets against which to compare but these nuances, while important in the day-to-day implementation, don’t blunt the general idea of APT – namely that the value of an asset (as determined by its return) depends on various factors and can only be judged in relation to the market as a whole.

The CAPM differs primarily from APT by its use of a single factor (a single ) to capture the systemic market risk. This aspect of the CAPM means that it assumes markets are perfectly efficient. It isn’t as accurate but it is much easier to use and this one feature explains its staying power.

One final note, the devil really is in the details for much of this work. In particular, it doesn’t seem as if there is a well-known discussion of the numerical stability of these results. Given that the linear-regressions (typically multi-variate) are used to determine the betas and, consequently, the risk premiums, there seems to be room to determine just how much additional risk is buried within the algorithm. But that is a blog for another day.